PAUL SCHERRER INSTITUT

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Introduction to Computed Tomography

Part V: Sampling and noise

Outline

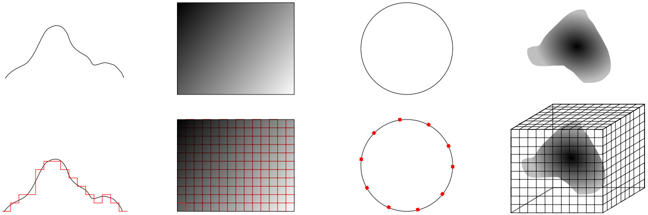
Lecture outline

- Different kinds of sampling
- Noise

Discretizing the reconstruction formula

The inversion formula is impractical since it would require infinite amount of equations to solve.

- The projections are digital images
  - Intensity sampling [bits/pixel]
  - Spatial sampling [pixels/mm]
- The rotation is done in steps
- The reconstruction is done on a finite matrix

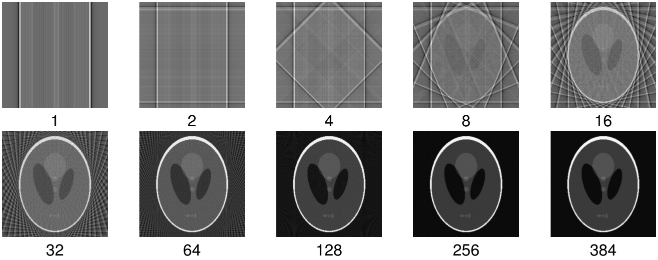


How many projections are needed?

The number of projections is determined by the sampling theorem [Buzug, 2008].

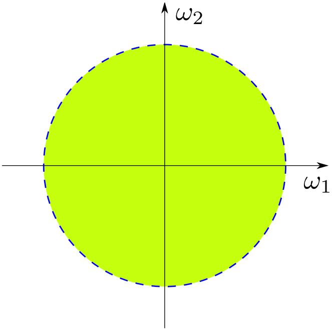
$$N_{projections} = \frac{\pi}{2} N_u$$

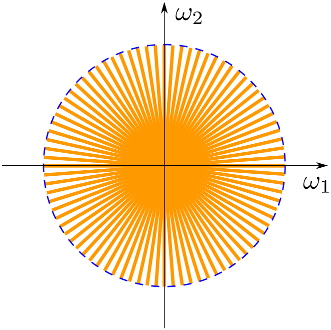
N_u Number of pixels in the direction perpendicular to the axis of rotation.



Intuitive proof of the sampling theorem

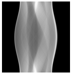
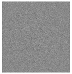
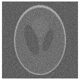
Basic idea The unit circle in the Fourier domain must be filled.





Noise

Noise is a statistical phenomenon.

$\mathcal{R}^{-1}\Big\{$  $\Big\} + \mathcal{R}^{-1}\Big\{$  $\Big\} = \mathcal{R}^{-1}\Big\{$  $\Big\} \rightarrow$ 

Noise sources:

- Noise induced by the radiation source.
- Thermal noise from the electronics.
- Algorithmic, rounding errors, interpolation model etc.

Dose and noise

The noise level of a slice is directly connected to the dose used.

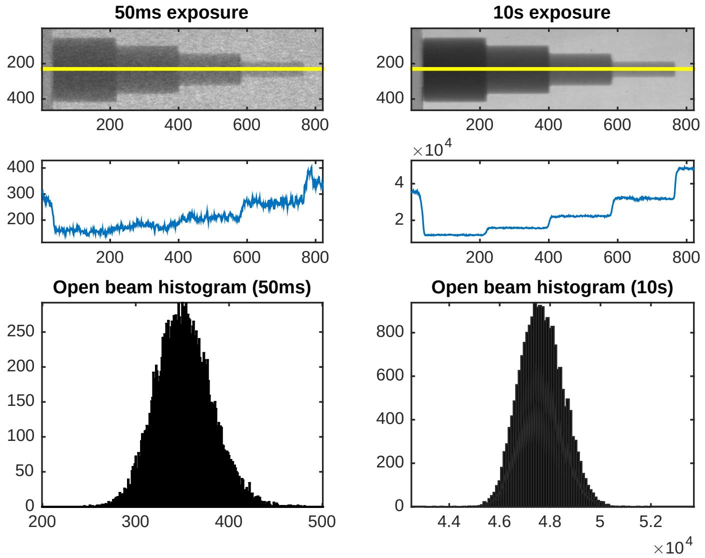
Definition

$$Dose = Flux \times Time$$

The signal to noise ratio can be improved by increasing

- the beam intensity,
- the exposure time,
- the number of projections,
- detector exchange.

Profiles with Poison noise



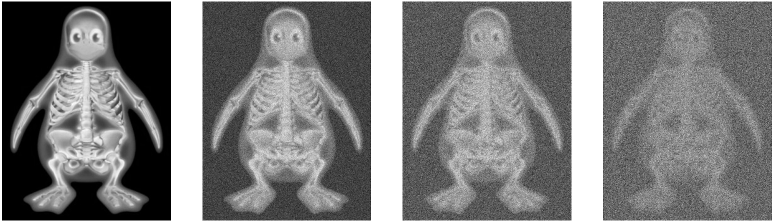
Signal to noise ratio

A metric to describe noise strength

$$SNR = \frac{\mu_{image}}{\sigma_{image}} \quad (1)$$

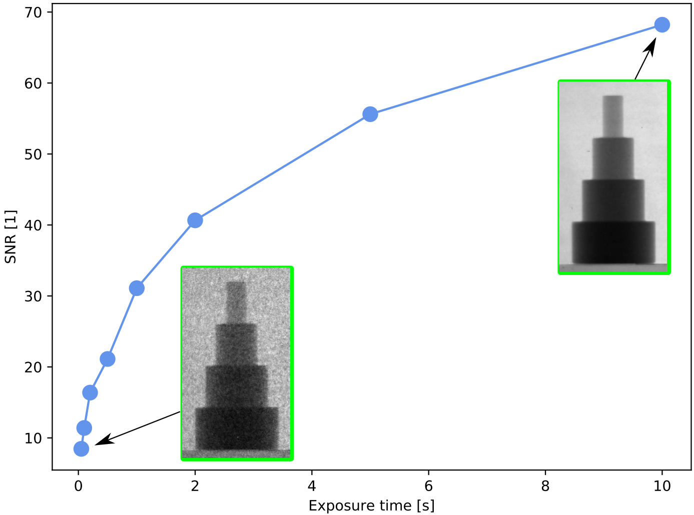
$$SNR_{db} = 20 \log \frac{\mu_{image}}{\sigma_{image}} \quad (2)$$

- Select a region
- Compute average intensity
- Compute std deviation
- Apply eqns 1 or 2



SNR = ∞ | SNR = 5 | SNR = 2 | SNR = 1

SNR for different exposure times



SNR for Poisson noise: $SNR = \frac{\mu}{\sigma} = \frac{\lambda}{\sqrt{\lambda}} = \sqrt{\lambda} \sim \sqrt{t}$

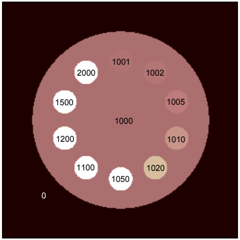
What influences the contrast?

$$C_{slice}\, W_{sample} \sim C_{projection}\, N_{projections}$$

- C_{slice} Slice contrast
- $C_{projection}$ Projection contrast (Open beam - darkest region)
- $N_{projections}$ Number of projections
- W_{sample} Largest width of the sample in pixels

Contrast experiment

The phantom

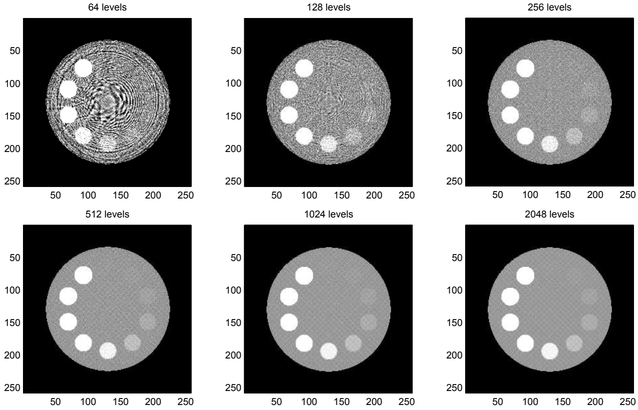


Parameters

- w=192
- $N_{projections}$=288
- $C_{projection}$=6, 7, 8, 9, 10,11, 12, 13 bits
- Contrast ratio: 1000:1, . . . , 1:2
- Noise free

What can be seen?

Changing projection contrast with constant number of projections



The reconstruction noise decrease with increasing dynamics

Summary

- Digital images are digitized on many levels.
- The number of projections is important for the image quality.
- The neutron flux and exposure time affect the SNR.
- A well utilized gray-level dynamics is important.

References I

Buzug, T. (2008).
Introduction to Computed Tomography: From photon statistics to modern cone-beam CT.
Springer.