PAUL SCHERRER INSTITUT

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Anders Kaestner :: Paul Scherrer Institut

Introduction to Computed Tomography

Part II: Radiography and primitive computed tomography

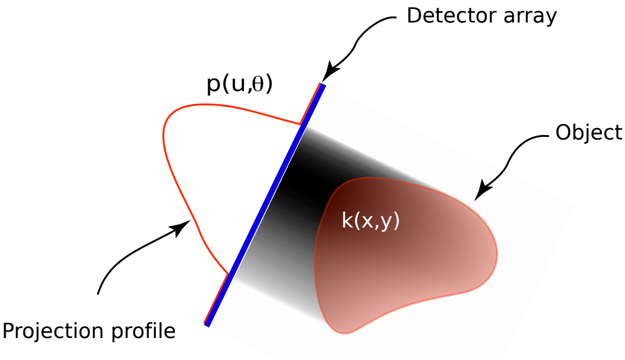
Outline

Learning objectives

- Understand the image formation process for radiography.
- Motivating the use of computed tomography.

Transmission image – the projection

A ray illuminates a semi-transparent medium



Transmission imaging – Radiography

A ray penetrating a medium is attenuated according to Beer-Lamberts law The intensity is attenuated in the medium according to

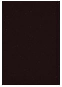
$$I = I_0 e^{-\int_L k(x,y)\,dl}$$

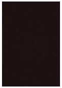
- I - Intensity behind the sample
- I_0 - Incident intensity
- k - Attenuation coefficient,
  - μ - Linear attenuation coefficient X-rays
  - Σ - Macroscopic cross-section for neutrons
- L - Line through the sample.

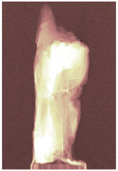
Computing an attenuation image

From Beer-Lamberts law we get

$$p = -\log\left(\frac{r - r_{DC}}{r_{OB} - r_{DC}}\right) = -\log\left(\frac{\square - \square}{\square - \square}\right) =$$

 − 

 − 

= 

p Normed projection

r Measured radiogram

r_{DC} Dark current image (removes noise floor)

r_{OB} Open beam image, measured I_0

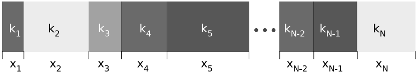
Each pixel represent the line integral $\int_L k(x)dx$ through the sample.

Generalized attenuation law 1D

Piecewise constant sample

Few discrete regions

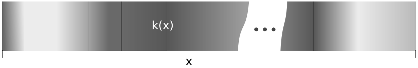
$$I = I_0\, e^{-\sum_{i=1}^{N} k_i\, x_i}$$



Continuous samples

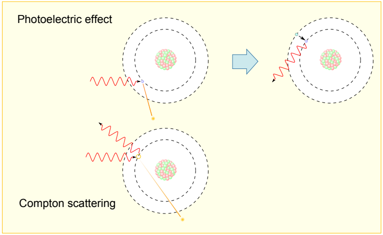
Let $x_i = \Delta x$ and $\Delta x \rightarrow 0$

$$I = I_0\, e^{-\int_L k(x)\, dx}$$



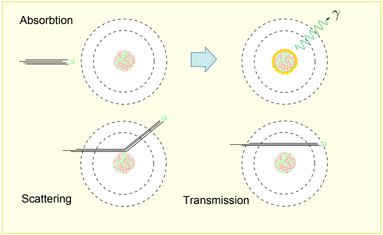
Different sources to illuminate the sample

X-rays



- Electromagnetic radiation.
- Interaction with the electron shells.

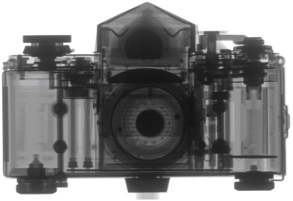
Neutrons



- Neutral particle beam.
- Interaction with the nucleus.

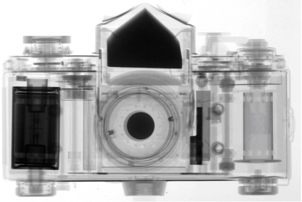
Attenuation coefficients

X-rays at 150keV



Group → / ↓Period	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
1	H 0.02																	He 0.02
2	Li 0.06	Be 0.22											B 0.28	C 0.27	N 0.11	O 0.16	F 0.14	Ne 0.17
3	Na 0.13	Mg 0.24											Al 0.38	Si 0.33	P 0.25	S 0.30	Cl 0.23	Ar 0.20
4	K 0.14	Ca 0.26	Sc 0.48	Ti 0.73	V 1.04	Cr 1.29	Mn 1.32	Fe 1.57	Co 1.78	Ni 1.96	Cu 1.97	Zn 1.64	Ga 1.42	Ge 1.33	As 1.50	Se 1.23	Br 0.90	Kr 0.73
5	Rb 0.47	Sr 0.86	Y 1.61	Zr 2.47	Nb 3.43	Mo 4.29	Tc 5.06	Ru 5.71	Rh 6.08	Pd 6.13	Ag 5.67	Cd 4.84	In 4.31	Sn 3.98	Sb 4.28	Te 4.06	I 3.45	Xe 2.53
6	Cs 1.47	Ba 2.73		Hf 19.70	Ta 25.47	W 30.49	Re 34.47	Os 37.92	Ir 39.01	Pt 38.61	Au 35.94	Hg 25.88	Tl 23.23	Pb 22.81	Bi 20.28	Po 20.22	At -	Rn 9.77
7	Fr -	Ra 11.80		Rf -	Db -	Sg -	Bh -	Hs -	Mt -	Ds -	Rg -	Uub -	Uut -	Uuq -	Uup -	Uuh -	Uus -	Uuo -
Lanthanides				La 5.04	Ce 5.79	Pr 6.23	Nd 6.46	Pm 7.33	Sm 7.68	Eu 5.66	Gd 8.69	Tb 9.46	Dy 10.17	Ho 10.17	Er 11.70	Tm 12.49	Yb 9.32	Lu 14.07
Actinides				Ac 24.47	Th 28.95	Pa 39.65	U 49.08	Np -	Pu -	Am -	Cm -	Bk -	Cf -	Es -	Fm -	Md -	No -	Lr -

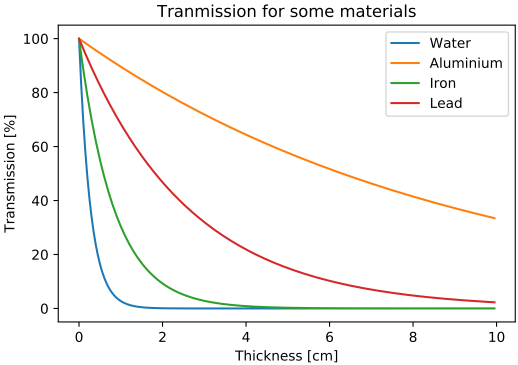
Thermal neutrons

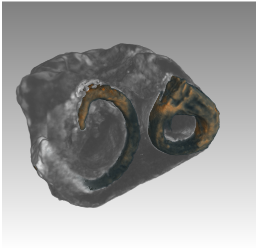


Group → / ↓Period	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
1	H 3.44																	He 0.02
2	Li 3.30	Be 0.79											B 101.6	C 0.56	N 0.43	O 0.17	F 0.20	Ne 0.10
3	Na 0.09	Mg 0.15											Al 0.1	Si 0.11	P 0.12	S 0.06	Cl 1.33	Ar 0.03
4	K 0.06	Ca 0.08	Sc 2.00	Ti 0.60	V 0.72	Cr 0.54	Mn 1.21	Fe 1.19	Co 3.92	Ni 2.05	Cu 1.07	Zn 0.35	Ga 0.49	Ge 0.47	As 0.67	Se 0.73	Br 0.24	Kr 0.61
5	Rb 0.08	Sr 0.14	Y 0.27	Zr 0.29	Nb 0.40	Mo 0.52	Tc 1.76	Ru 0.58	Rh 10.88	Pd 0.78	Ag 4.04	Cd 115.1	In 7.58	Sn 0.21	Sb 0.30	Te 0.25	I 0.23	Xe 0.43
6	Cs 0.29	Ba 0.07		Hf 4.99	Ta 1.49	W 1.47	Re 6.85	Os 2.24	Ir 30.46	Pt 1.46	Au 6.23	Hg 16.21	Tl 0.47	Pb 0.38	Bi 0.27	Po -	At -	Rn -
7	Fr -	Ra 0.34		Rf -	Db -	Sg -	Bh -	Hs -	Mt -	Ds -	Rg -	Uub -	Uut -	Uuq -	Uup -	Uuh -	Uus -	Uuo -
Lanthanides				La 0.52	Ce 0.14	Pr 0.41	Nd 1.87	Pm 5.72	Sm 171.47	Eu 94.58	Gd 1479.0	Tb 0.93	Dy 32.42	Ho 2.25	Er 5.48	Tm 3.53	Yb 1.40	Lu 2.75
Actinides				Ac -	Th 0.59	Pa 8.46	U 0.82	Np 9.80	Pu 50.20	Am 2.86	Cm -	Bk -	Cf -	Es -	Fm -	Md -	No -	Lr -

[Sears, 1992]

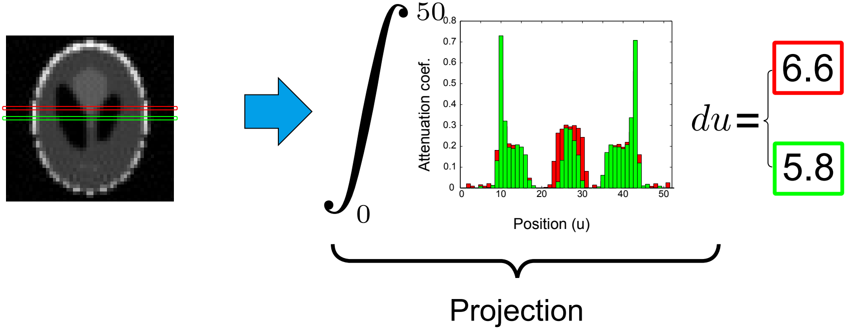
Some attenuation examples for neutrons





Neutron tomography of fist-sized lead canon ball from the battle of Bossworth (1485AD)

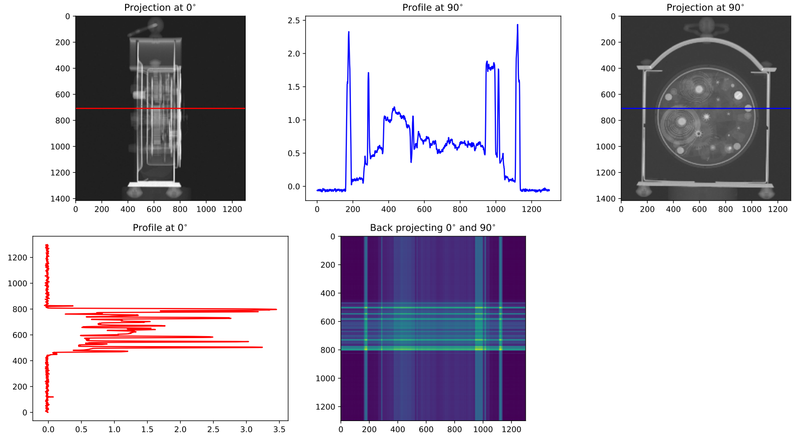
Limitation of the radiography



- Great local changes buried in the sum of bulk
- Depth position can’t be determined

Stereography

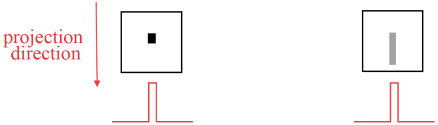
Use two projections at 90° to get depth information



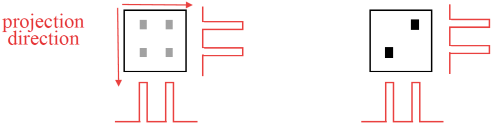
- Provides some depth information
- Still a lot of guessing

The solution is not unique

Single projection → several solutions



Two projections → several solutions



A unique solution would exist only for an infinite number of noiseless continuous projections

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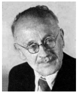
What is tomography?

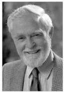
- A method to capture three-dimensional images.
- An indirect method using projections (radiograms) to reconstruct the inner structure of a sample.
- Free translation is slice imaging
  from Greek:

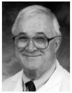
  Tomos – ’a section’ or ’a cutting’
  Graph – write

History

- 1917 **Radon** developed the foundation for the inversion required by tomography.
- 1956 **Bracewell** the relationships between Fourier transform and Radon transform.
- 1963 First applications to medical tomography. **Kuhl** obtained first backprojection. **Cormack** applied Radon's results to radiograms.
- 1970 Publication of the first CT image.
- 1970-1973 **Cormack** & **Hounsfield** first CT scanner.
- 1979 **Cormack** & **Hounsfield** the Nobel prize in Medicine.


J. Radon (1887–1956)

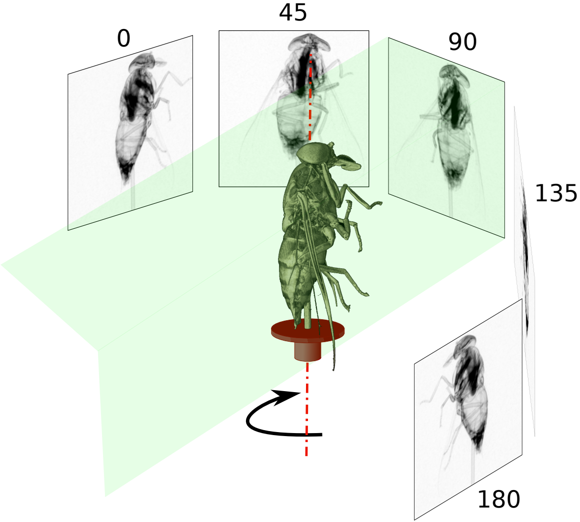

R. Bracewell (1921–2007)


D Kuhn (1929–2017)


A. Cormack (1924–1998)


Sir G.N. Hounsfield (1919–2004)

Inspecting the sample from different views



A first attempt to reconstruction: Algebraic solution

Observations

$$\begin{array}{ccl} 2 & 3 & \to 5 \\ 1 & 4 & \to 5 \\ \downarrow & \downarrow & \\ 3 & 7 & \end{array}$$

Equation system

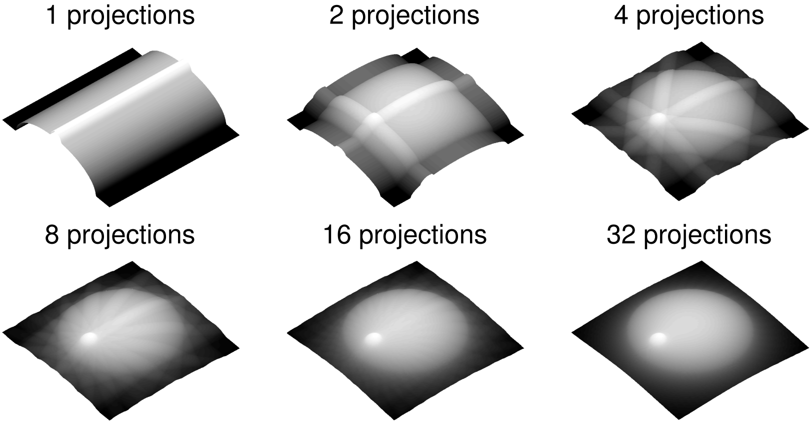
$$\begin{aligned} a_{11}x_1 + a_{12}x_2 &= y_1 \\ a_{21}x_3 + a_{22}x_4 &= y_2 \\ a_{11}x_1 + a_{21}x_3 &= y_3 \\ a_{12}x_2 + a_{22}x_4 &= y_4 \\ &\vdots \end{aligned}$$

$$\Rightarrow A\,x = y$$

Solve the equation system for x

Many equations, sparse matrix A, no unique solution...

A first attempt to reconstruction: Back-projection



The solution is too smooth. . . something is missing!!!

Summary

- Radiography to inspect interiors of opaque objects.
- Different sources and the attenuation law.
- First motivating steps towards computed tomography.

References I

Sears, V. (1992).
Neutron scattering lengths and cross sections.
Neutron News, 3(3):26–37.